

1.1 What is a Signal

We are all immersed in a sea of signals. All of us from the smallest living unit, a cell, to the most complex living organism(humans) are all time time receiving signals and are processing them. Survival of any living organism depends upon processing the signals appropriately. What is signal? To define this precisely is a difficult task. Anything which carries information is a signal. In this course we will learn some of the mathematical representations of the signals, which has been found very useful in making information processing systems. Examples of signals are human voice, chirping of birds, smoke signals, gestures (sign language), fragrances of the flowers. Many of our body functions are regulated by chemical signals, blind people use sense of touch. Bees communicate by their dancing pattern. Some examples of modern high speed signals are the voltage charger in a telephone wire, the electromagnetic field emanating from a transmitting antenna, variation of light intensity in an optical fiber. Thus we see that there is an almost endless variety of signals and a large number of ways in which signals are carried from one place to another place.

In this course we will adopt the following definition for the signal: A signal is a real (or complex) valued function of one or more real variable(s).

When the function depends on a single variable, the signal is said to be one-dimensional. A speech signal, daily maximum temperature, annual rainfall at a place, are all examples of a one dimensional signal. When the function depends on two or more variables, the signal is said to be multidimensional. An image is representing the two dimensional signal, vertical and horizontal coordinates representing the two dimensions. Our physical world is four dimensional(three spatial and one temporal).

1.2 What is signal processing

By processing we mean operating in some fashion on a signal to extract some useful information. For example when we hear same thing we use our ears and auditory path ways in the brain to extract the information. The signal is processed by a system. In the example mentioned above the system is biological in nature. We can use an electronic system to try to mimic this behavior. The signal processor may be an electronic system, a mechanical system or even it might be a computer program.

The word digital in digital signal processing means that the processing is done either by a digital hardware or by a digital computer.

1.3 Analog versus digital signal processing

The signal processing operations involved in many applications like communication systems, control systems, instrumentation, biomedical signal processing etc can be implemented in two different ways

- (1) Analog or continuous time method and
- (2) Digital or discrete time method.

The analog approach to signal processing was dominant for many years. The analog signal processing uses analog circuit elements such as resistors, capacitors, transistors, diodes etc. With the advent of digital computer and later microprocessor, the digital signal processing has become dominant now a days.

The analog signal processing is based on natural ability of the analog system to solve differential equations the describe a physical system. The solution are obtained in real time. In contrast digital signal processing relies on numerical calculations. The method may or may not give results in real time. The digital approach has two main advantages over analog approach

- (1) Flexibility: Same hardware can be used to do various kind of signal processing operation,while in the core of analog signal processing one has to design a system for each kind of operation.
- (2) Repeatability: The same signal processing operation can be repeated again and again giving same results, while in analog systems there may be parameter variation due to change in temperature or supply voltage.

The choice between analog or digital signal processing depends on application. One has to compare design time,size and cost of the implementation.

1.4 Classification of signals

As mentioned earlier, we will use the term signal to mean a real or complex valued function of real variable(s). Let us denote the signal by $x(t)$. The variable t is called independent variable and the value x of t as dependent variable. We say a signal is continuous time signal if the independent variable t takes values in an interval.

For example $t \in (-\infty, \infty)$, or $t \in [0, \infty]$ or $t \in [T_0, T_1]$

The independent variable t is referred to as time,even though it may not be actually time. For example in variation if pressure with height t refers above mean sea level.

When t takes a vales in a countable set the signal is called a discrete time signal. For example

$t \in \{0, T, 2T, 3T, 4T, \dots\}$ or $t \in \{\dots - 1, 0, 1, \dots\}$ or $t \in \{1/2, 3/2, 5/2, 7/2, \dots\}$ etc.

For convenience of presentation we use the notation $x[n]$ to denote discrete time signal.

Let us pause here and clarify the notation a bit. When we write $x(t)$ it has two meanings. One is value of x at time t and the other is the pairs $(x(t), t)$ allowable value of t . By signal we mean the second interpretation. To keep this distinction we will use the following notation: $\{x(t)\}$ to denote the continuous time signal. Here $\{x(t)\}$ is short notation for $\{x(t), t \in I\}$ where I is the set in which t takes the value. Similarly for discrete time signal we will use the notation $\{x[n]\}$, where $\{x[n]\}$ is short for $\{x[n], n \in I\}$. Note that in $\{x(t)\}$ and $\{x[n]\}$ are dummy variables ie. $\{x[n]\}$ and $\{x[t]\}$ refer to the same signal. Some books use the notation $x[\cdot]$ to denote $\{x[n]\}$ and $x[n]$ to denote value of x at time n · $x[n]$ refers to the whole waveform, while $x[n]$ refers to a particular value. Most of the books do not make this distinction clean and use $x[n]$ to denote signal and $x[n_0]$ to denote a particular value.

As with independent variable t , the dependent variable x can take values in a continuous set or in a countable set. When both the dependent and independent variable take value in intervals, the signal is called an analog signal. When both the dependent and independent variables take values in countable sets (two sets can be quite different) the signal is called Digital signal. When we use digital computers to do processing we are doing digital signal processing. But most of the theory is for discrete time signal processing where default variable is continuous. This is because of the mathematical simplicity of discrete time signal processing. Also digital signal processing tries to implement this as closely as possible. Thus what we study is mostly discrete time signal processing and what is really implemented is digital signal processing.

Exercise:

1. Give examples of continuous time signals.
2. Give examples of discrete time signals.
3. Give examples of signal where the independent variable is not time (one-dimensional).
4. Give examples of signal where we have one independent variable but dependent variable has more than one dimension. (This is sometimes called vector valued signal or multichannel signal).
5. Give examples of signals where dependent variable is discrete but independent variable are continuous.

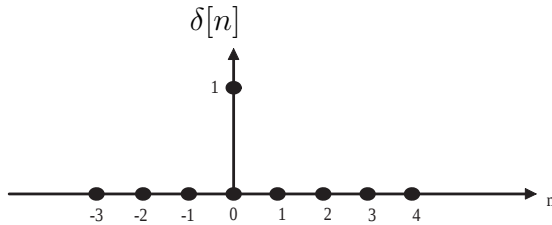
1.5 Elementary signals

There are several elementary signals that feature prominently in the study of digital signals and digital signal processing.

(a) Unit sample sequence $\delta[n]$: Unit sample sequence is defined by

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

Graphically this is as shown below.

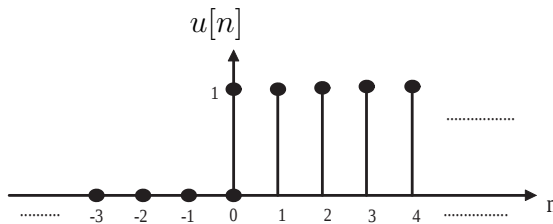


Unit sample sequence is also known as impulse sequence. This plays role akin to the impulse function $\delta(t)$ of continuous time. The continuous time impulse $\delta(t)$ is purely a mathematical construct while in discrete time we can actually generate the impulse sequence.

(b) Unit step sequence $u[n]$: Unit step sequence is defined by

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Graphically this is as shown below

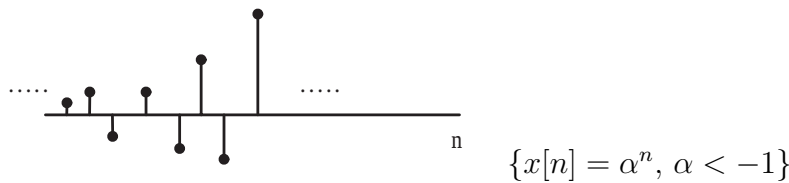
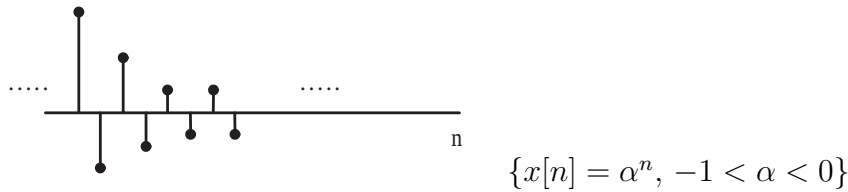
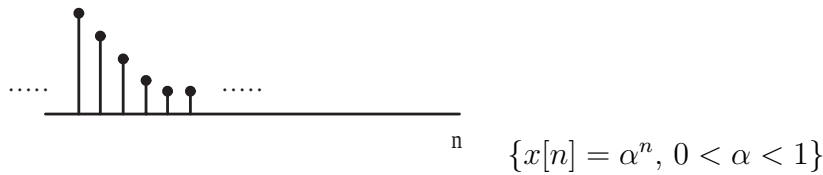
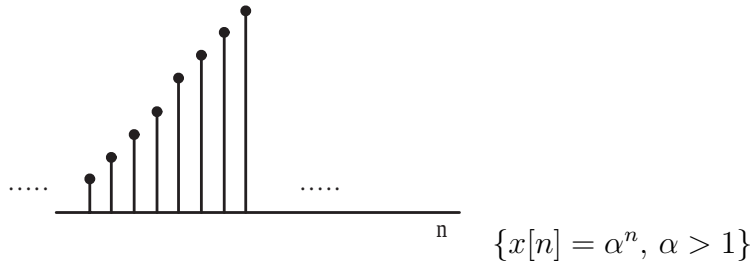


(c) Exponential sequence: The complex exponential signal or sequence $x[n]$ is defined by

$$x[n] = C \alpha^n$$

where C and α are, in general, complex numbers. Note that by writing $\alpha = e^\beta$, we can write the exponential sequence as $x[n] = c e^{\beta n}$.

Real exponential signals: If C and α are real, we can have one of the several type of behavior illustrated below



if $|\alpha| > 1$ the magnitude of the signals grows exponentially, while if $|\alpha| < 1$, we have decaying exponential. If α is positive all terms of $\{x[n]\}$ have same sign, but if α is negative the sign of terms in $\{x[n]\}$ alternates.

(d) Sinusoidal Signal: The sinusoidal signal $\{x[n]\}$ is defined by

$$x[n] = A \cos(w_0 n + \phi)$$

Euler's relation allows us to relate complex exponentials and sinusoids.

$$e^{j w_0 n} = \cos w_0 n + j \sin w_0 n$$

and

$$A \cos(w_0 n + \phi) = \frac{A}{2} e^{j\phi} e^{j w_0 n} + \frac{A}{2} e^{-j\phi} e^{-j w_0 n}$$

The general discrete time complex exponential can be written in terms of real exponential and sinusoidal signals. Specifically if we write C and α in polar for $C = |C|e^{j\theta}$ and $\alpha = |\alpha|e^{j w_0}$ then

$$C \alpha^n = |C| |\alpha|^n \cos(w_0 n + \theta) + j |C| |\alpha|^n \sin(w_0 n + \theta)$$

Thus for $|\alpha| = 1$, the real and imaginary parts of a complex exponential sequence are sinusoidal. For $|\alpha| < 1$, they correspond to sinusoidal sequence multiplied by a decaying exponential, and for $|\alpha| > 1$ they correspond to sinusoidal sequence multiplied by a growing exponential.

1.6 Generating Signals with MATLAB

MATLAB, acronym for MATrix LABoratory has become a very popular software environment for complex based study of signals and systems. Here we give some sample programmes to generate the elementary signals discussed above. For details one should consider MATLAB manual or read help files. In MATLAB, `ones(M,N)` is an M-by-N matrix of ones, and `zeros(M,N)` is an M-by-N matrix of zeros. We may use those two matrices to generate impulse and step sequence.

The following is a program to generate and display impulse sequence.

```
>> % Program to generate and display impulse response sequence
>> n = -49 : 49;
>> delta = [zeros(1, 49), 1, zeros(1, 49)];
>> stem(n, delta)
```

Here `>>` indicates the MATLAB prompt to type in a command, `stem(n,x)` depicts the data contained in vector `x` as a discrete time signal at time values defined by `n`. One can add title and label the axes by suitable commands.

To generate step sequence we can use the following program

```
>> % Program to generate and display unit step function
>> n = -49 : 49;
>> u = [zeros(1, 49), ones(1, 50)];
>> stem(n, u);
```

We can use the following program to generate real exponential sequence

```
>> % Program to generate real exponential sequence
>> C = 1;
>> alpha = 0.8;
>> n = -10 : 10;
>> x = C * alpha.^ n
>> stem(n, x)
```

Note that, in their program, the base `alpha` is a scalar but the exponent is a vector, hence use of the operator `.^` to denote element-by-element power.

Exercise: Experiment with this program by changing different values of `alpha` (real). Values of `alpha` greater than 1 will give growing exponential and less than 1 will give decaying exponentials.